RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY-JUNE 2013

FIRST YEAR

Date : 24/05/2013 **MATHEMATICS (General) [For Economics]**

Time : 11am – 2pm Paper : II Full Marks : 75

[Use separate Answer Books for each group]

Group-A

1. Answer **any two** : 2×10

a) i) If a function has a finite derivative at a point, prove that it is continuous at that point. Show also by an example that the converse is not necessarily true. 3+2

ii) Find $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$. 5

b) i) State and prove Lagrange's mean value theorem. 6

ii) Are the conditions of Rolle's theorem satisfied for the function $g(x) = x^2$ in $2 \le x \le 3$? 4

c) i) Express $\int_a^b e^x dx$ as the limit of a sum and evaluate it. 5

ii) Find the reduction formulae for $\int \sin^m x \cos^n x \, dx$ (where *m*, *n* are +ve integers, greater than 1) and evaluate $\int \sin^4 x \cos^2 x \, dx$. 5

2. Answer **any four** : 4×5

a) i) If $f(x) = 2|x| + |x-2|$, find $f'(1)$. 3

ii) Write the Geometrical interpretation of Rolle's theorem. 2

b) Show that the maximum rectangle inscribed in a circle is a square. 5

c) Find the maximum value of $f(x) = x^{\frac{1}{x}}$. 5

d) Integrate: $\int \frac{\sin x}{\sin x - \cos x} dx$. 5

e) Integrate by definition: $\int_0^1 2^x dx$. 5

f) State Libnitz's theorem and use it to prove that if $y = \sin(m \sin^{-1} x)$, then $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. 1+4

Group-B

Unit - I

3. Answer **any three** : 3×5

a) Find the rank of the following matrix: 5

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 5 & 9 & 10 \end{pmatrix}$$

b) Obtain the fully reduced normal form of the matrix: 5

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

c) What is elementary matrices? Express the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{bmatrix}$ as a product of elementary matrices. 5

d) Solve, if possible

$$x + 2y + z - 3w = 1$$
$$2x + 4y + 3z + w = 3$$
$$3x + 6y + 4z - 2w = 4$$

5

e) Determine the condition for which the following system of equation has
(i) Only one solution, (ii) No solution, (iii) Many solution 5

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x + ay + 3z = b + 1$$

4. Answer **any <u>two</u>** : 2×5

a) Define Δ and E operators.
Prove that $\Delta[Ef(x)] = E[\Delta f(x)]$. 1+1+3

b) Show that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$. 5

c) Formulate the difference equation where given, $u_x = (C + Dx) \cdot 3^x$. 5

d) Solve the difference equation $u_{x+2} - 5u_{x+1} + 6u_x = 2x^2 + 3$, where the interval of difference is unity. 5

<u>Unit - II</u>

5. Answer **any <u>two</u>** : 2×5

a) i) Determine the order and degree of the following differential equation 2

$$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = x^2.$$

ii) Eliminate the parameters a, b from the following primitive to construct a differential equation
$xy = ae^x + be^{-x}$ 3

b) What is exact differential equation?
Show that the given differential equation is exact and hence solve it.
$(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$. 5

c) Solve: $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$. 5

d) Solve: $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 5